Fun With HyperCuber

This section is basically a tutorial. It shows some of the more interesting things I have discovered using HyperCuber.

When you start up HyperCuber, you are presented with a weird-looking object, a “cube within a cube.” This is perhaps a little dizzying for starters, so try setting both perspective parameters ([3:P] and [4:P]) and all the angles to 0. Better? It should look like this (Rather than using the default hypercube, which gets confusing because of its colors I’ll start the tutorial using a hypercube created using the Create N-Cube command):

You’re probably more familiar with this object than you were with the hypercube: it’s called a square. Now, the entire hypercube is still there, but every single line segment lies on or behind or in front of one of those four, so it looks a little simpler.

Ready for another dimension? Try cranking the 3D perspective [3:P] up to 6. It should look like this:

Rotate it a little (change [3:1] to 30) and it looks like this:

Here we have a cube. Again, this is still that hypercube, but since we've eliminated 4D perspective, all the line segments are lined up on top of the twelve you see above.

For the final step, turn up [4:P] (four-dimensional perspective) to 7 and turn [4:3] to 90. I am now using color because the color is more meaningful than it was in 2D.

We're back to the original hypercube. Note the similarity between this and Figure 2. This is a (green) cube within a (blue) cube, with the corners of the inner and outer cubes connected by (red) line segments; Figure 2 is a square within a square, with the corners connected by line segments. Figure 2 is a cube viewed face on; Figure 4 is a hypercube viewed face-on.

Perhaps you are shrieking, "How can you say that Figure 4 is face on, when we're clearly looking at one of the corners!?" The answer is that four-dimensionally we are looking at it face-on, but we are looking at the three-dimensional projection of it askew.

It is vital that you understand the difference between a higher-dimensional object and its three-dimensional projection. You are never looking at the hypercube itself when you look at your screen. Instead, you are looking at a three-dimensional projection of it. In fact (if stereo is turned off), you are looking at a two-dimensional projection of the three-dimensional projection of a hypercube!

It is also important that you realize that all objects which HyperCuber displays are fixed and rigid in their native dimension. That is, the hypercube never moves, never contorts, and never rotates. When it seems to rotate, it’s really you, orbiting the three-dimensional projection. When it seems to contort, it’s also you, orbiting the four-dimensional hypercube, a movement which can have very strange effects on the three-dimensional projection. Note that the three-dimensional projection does contort— this is a normal part of projection, and happens not only when you project four-dimensional object onto three-space, but also when you project three-dimensional objects onto two-space. For example, turn down the four-dimensional perspective again, and change [3:1] a bit:

Now you, the three-dimensional being, understand that these are just different views of the same object, and it looks like it’s a rigid cube. But think about it from a two-dimensional perspective for a minute. The three images are really quite different, and hardly seem rigid, viewed as images in a plane. Similarly, when you start changing the four-dimensional view of a rigid object (using [4:1], [4:2] and [4:3]), it will look like the object’s doing some really bizarre stuff. But keep in mind: the object is rigid in four-space; it’s just the projection process that makes it look like it’s contorting. Similarly, a five-dimensional object is rigid in five-space, but its four-dimensional, three-dimensional, and two-dimensional projections contort as the viewpoints and perspectives change.

Let’s look at a four-dimensional rotation analogous to Figure 5; we turn up [4:P] to 7 set all angles to 0 except [3:1] which we set to 3, and rotate using [4:3]. Figure 6 shows some of the frames:

It is clear what is happening to the three-dimensional projection from the frames above— the top green “face” (which looks like an inverted decapitated pyramid) is moving gradually downward until at last it becomes the bottom face (which looks like an upright decapitated pyramid).

Close examination of this situation proves very enlightening. Remember, a hypercube is an object composed of eight cubes. It’s not too hard to see two of them in the fifth frame; one is green and the other is blue.   But where are the rest? Before we answer that, let’s look at what rotating a cube does to its two-dimensional projection. It turns out that simple rotations of a cube, and of a hypercube, tend to just permute the faces. For instance, a cube rotates like this:

The blue face which is on the bottom in the first frame is in the front in the fifth frame; the face in front in the first frame ends on the top in the fifth frame, the green top goes to the back, and the back goes to the bottom. The left and right faces (perpendicular to the axis of rotation) are contorted in this two-dimensional projection, but do not actually move to a new location; the left face at the end is the same face which was on the left at the beginning (though the edges are permuted).

Now let's identify all eight faces of the hypercubes. The two we have found are the inner and outer cubes. The other six correspond to the faces of these cubes. For instance, take the top face of the inner cube and the top face of the outer cube in the first frame of Figure 6, and connect the corners of these faces with lines. What you have is the green six-sided solid, the kind I was calling a "decapitated pyramid" before. This is in fact one of the faces of the hypercube, and the other five correspond to the bottom faces and the side faces of the inner and outer cubes. This face doesn't look too much like a cube (its faces are different shapes and sizes, for instance), but that's just a result of the projection of the hypercube onto three-space. For an analogy, look at the cubes in Figure 7 (remembering to look at them as two-dimensional objects, rather than "understanding" them as three-dimensional objects)— their faces don't look much like squares either; the sides aren't parallel, some are squished to the sides, some lines are diagonal and others are horizontal. The same thing is happening to the hypercube that happens to the cube when it's projected to the plane — the faces are getting distorted. A four-dimensional person could look at the three-dimensional projection of the hypercube, and understand what the real object looked like, just as you can look at the two-dimensional pictures of cubes above and understand them as three-dimensional objects. The three-dimensional projections of the hypercube are the sort of things such a creature would see when it unfolded its three-dimensional paper in the morning and started to read — its mind would fill in the extra dimension just as three-dimensional minds understand two-dimensional pictures as representations of three-dimensional scenes. We three-dimensional beings, hampered by a lifetime of experience in a three-dimensional world, cannot directly interpret the three-dimensional projection as a projection of a four-dimensional object. The best we can do is compare it to two-dimensional projections of three-dimensional objects and draw analogies.

Now the fun really begins. Note that the four-dimensional rotation of the hypercube, strange though it looks, is fundamentally exactly the same as the three-dimensional rotation of the cube! Remember, we noticed earlier that the inner face in frame 5 of Figure 6 (the green cube) has become the bottom face by the end. Similarly, the bottom face becomes the outer face, the outer face becomes the top face, and the top face becomes the inner face. Very similar to the rotating cube.... Furthermore, the left, right, front, and back faces contort but do not permute with the other faces; this is similar to the top and bottom faces of the cube. There are more faces which stay “fixed” in a hypercube because rotational axes are planes, rather than lines, in four-space. More of the faces (four of them, in fact) lie on the axis of rotation. If you compare Figure 6 and Figure 7, you will notice a remarkable number of parallels.

This concludes the tutorial. I have described only a little of what I have discovered about hypercubes, and it is easily extended to other objects and higher dimensions. I leave the rest to you. If you can, you should use the stereo mode to enhance comprehension, especially if you’re viewing a monochrome object; it is very easy to misinterprete a four-dimensional rotation as a three-dimensional rotation if you don’t have the aid of colors or stereoscopic vision. If you can’t tell what something is doing while you are rotating it four-dimensionally, try stopping it and circling it a few times three-dimensionally using [3:1] and [3:2] (use only [3:1] if you’re viewing in stereo). Remember, the best way to understand the fourth dimension is to compare it to the third— the two have virtually everything in common if you look closely. Have fun!